

Unit 1

Binomial coefficients

$${}_n C_r = \binom{n}{r} \quad (1)$$

$$\binom{n}{r} = \binom{n}{n-r} \quad (2)$$

$$\binom{n}{r} = \frac{n!}{(n-r)!r!} \quad (3)$$

Partial Fractions

$$\frac{cx + d}{(ax + b)^2} = \frac{A}{ax + b} + \frac{C}{(ax + b)^2} \quad (4)$$

Limits

$$\lim_{h \rightarrow 0} \left(\frac{\sin h}{h} \right) = 1 \quad (5)$$

$$\lim_{h \rightarrow 0} \left(\frac{\sin 2h}{h} \right) = 2 \quad (6)$$

$$\lim_{h \rightarrow 0} \left(\frac{\cos h - 1}{h} \right) = 0 \quad (7)$$

$$\lim_{h \rightarrow 0} \left(\frac{\cos h}{h} \right) = \infty \quad (8)$$

Differentiation rules

- $f'(x)$ only exists if it is the same, and exists, when approaching from both sides. (I.e. it won't exist where there is a break, or an abrupt angled change.)
- Critical points are where $f'(x) = 0$ or doesn't exist.
- Points of inflection are where $f''(x) = 0$.

$$f'(x) = \lim_{h \rightarrow 0} \left(\frac{f(x+h) - f(x)}{h} \right) \quad (9)$$

$$\frac{d}{dx} (f(x) + g(x)) = f'(x) + g'(x) \quad (10)$$

$$\frac{d}{dx} f(g(x)) = f'(g(x)) \cdot g'(x) \quad (11)$$

$$\frac{d}{dx} (f(x) \cdot g(x)) = f'(x) \cdot g(x) + f(x) \cdot g'(x) \quad (12)$$

$$\frac{d}{dx} \left(\frac{f(x)}{g(x)} \right) = \frac{f'(x) \cdot g(x) - f(x) \cdot g'(x)}{(g(x))^2} \quad (13)$$

Newton's method for approximating roots

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \quad (14)$$

Trig identities

$$\tan^2 x + 1 = \sec^2 x \quad (15)$$

$$\cos 2\theta = 1 - 2 \sin^2 \theta \quad (16)$$

Standard derivatives

$$\frac{d}{dx} \tan x = \sec^2 x \quad (17)$$

$$\frac{d}{dx} \operatorname{cosec} x = -\operatorname{cosec} x \cot x \quad (18)$$

$$\frac{d}{dx} \sec x = \sec x \tan x \quad (19)$$

$$\frac{d}{dx} \cot x = -\operatorname{cosec}^2 x \quad (20)$$

$$\frac{d}{dx} \ln a \cdot x = \frac{1}{x} \quad (21)$$

Note:

$$\frac{d}{dx} \ln(x+1) = \frac{1}{x+1} \quad (22)$$

$$\frac{d}{dx} e^x = e^x \quad (23)$$

$$\frac{d}{dx} e^{ax} = a \cdot e^{ax} \quad (24)$$

$$\frac{d}{dx}x.e^x = (x + 1).e^x \quad (25)$$

Standard integrals

$$\int (ax + b)^n dx = \frac{(ax + b)^{n+1}}{a.(n + 1)}, \quad n \neq -1 \quad (26)$$

$$\int e^{ax+b} dx = \frac{1}{a}e^{ax+b} + c \quad (27)$$

$$\int \frac{1}{ax + b} = \frac{1}{a} \ln |ax + b| + c \quad (28)$$

$$\int \sec^2(ax + b) dx = \frac{1}{a} \tan(ax + b) + c \quad (29)$$

Other important integrals

$$\int \cos(ax + b) dx = \frac{1}{a} \sin(ax + b) + c \quad (30)$$

$$\int \cos^2 \theta d\theta = \frac{1}{2}\theta + \frac{1}{4} \sin 2\theta + c \quad (31)$$

$$\int \sin^2 \theta d\theta = \frac{1}{2}\theta - \frac{1}{4} \sin 2\theta + c \quad (32)$$

These use things such as (16).

$$\int \frac{f'(x)}{f(x)} dx = \ln |f(x)| + c \quad (33)$$

Unit 2

Sequences

Arithmetic sequence:

$$u_n = a + (n - 1) \times d \quad (34)$$

Geometric sequence:

$$u_n = a \times r^{n-1} \quad (35)$$

Derivatives of inverse trig. functions

$$\frac{d}{dx} (f^{-1}(x)) = \frac{1}{f'(f^{-1}(x))} \quad (36)$$

$$\frac{dx}{dy} = \frac{1}{\frac{dy}{dx}} \quad (37)$$

$$\frac{d}{dx} (\sin^{-1}(x)) = \frac{1}{\sqrt{1-x^2}} \quad (38)$$

But:

$$\frac{d}{dx} (\sin^{-1}(2x)) = \frac{2}{\sqrt{1-4x^2}} \quad (39)$$

$$\frac{d}{dx} (\cos^{-1}(x)) = -\frac{1}{\sqrt{1-x^2}} \quad (40)$$

$$\frac{d}{dx} (\tan^{-1}(x)) = \frac{1}{1+x^2} \quad (41)$$

Common implicit derivatives

Function	Derivative wrt x
y	$\frac{dy}{dx}$
y^2	$2y \cdot \frac{dy}{dx}$
$x \cdot y$	$x \cdot \frac{dy}{dx} + y$
$x \cdot y^2$	$x \cdot 2y \cdot \frac{dy}{dx} + y^2$

(42)

Miscellanea

These few are sometimes needed, and not specified anywhere in the course, but are relatively easy to derive.

$$\sec^{-1}(x) = \cos^{-1}\left(\frac{1}{x}\right) \quad (43)$$

$$\operatorname{cosec}^{-1}(x) = \sin^{-1}\left(\frac{1}{x}\right) \quad (44)$$

$$\cot^{-1}(x) = \tan^{-1}\left(\frac{1}{x}\right) \quad (45)$$

For a parametric equation, parameter t :

$$\frac{d^2 y}{dx^2} = \frac{d}{dt} \left(\frac{dy}{dx} \right) \times \frac{1}{\frac{dx}{dt}} \quad (46)$$

$$\forall = \text{'for all'} \quad (47)$$

$$\exists = \text{'for some / there exists'} \quad (48)$$

Further Integration

$$\int \frac{1}{\sqrt{a^2 - x^2}} = \sin^{-1}\left(\frac{x}{a}\right) + c \quad (49)$$

$$\int \frac{1}{a^2 + x^2} = \frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right) + c \quad (50)$$

$$\int \sec(\theta) = \ln |\sec(\theta) + \tan(\theta)| + c \quad (51)$$

$$\int \cot(\theta) = \ln |\sin(\theta)| + c \quad (52)$$

$$\int f'g = [fg] - \int g'f + c \quad (53)$$

Sequences and Series

Arithmetic

$$U_n = a + (n - 1)d \quad (54)$$

$$S_n = \frac{n}{2} [2a + (n - 1)a] \quad (55)$$

$$\sum_{r=1}^n r = \frac{n(n+1)}{2} \quad (56)$$

Geometric

$$U_n = a \cdot r^{n-1} \quad (57)$$

$$S_n = \frac{a(1 - r^n)}{1 - r} \quad (58)$$

$$S_\infty = \frac{a}{1 - r} \quad (59)$$

$$\frac{1}{1 - x} = 1 + x + x^2 + x^3 + \dots \quad (60)$$

$$\lim_{n \rightarrow \infty} \left\{ \left(1 + \frac{1}{n} \right)^n \right\} = e = 2.7182818\dots \quad (61)$$

Unit 3

Vectors

Scalar product

$$a \cdot b = a_1 b_1 + a_2 b_2 + a_3 b_3 \quad (62)$$

$$a \cdot b = |a||b| \cos \theta \quad (63)$$

$$\cos \theta = \frac{a \cdot b}{|a||b|} \quad (64)$$

$$a \cdot b = 0 \text{ for perpendicular vectors} \quad (65)$$

$$a \cdot a = |a|^2 \quad (66)$$

Cross / Vector product

$$|u \times v| = |u||v| \sin \theta \quad (67)$$

$$(u \times v) \cdot u = 0 \quad (68)$$

$$(u \times v) \cdot v = 0 \quad (69)$$

$$a \times b = \underline{i}(a_2 b_3 - a_3 b_2) - \underline{j}(a_1 b_3 - a_3 b_1) + \underline{k}(a_1 b_2 - a_2 b_1) \quad (70)$$

$$a \times b = -(b \times a) \quad (71)$$

$$a \cdot (b \times c) = a_1(b_2 c_3 - b_3 c_2) - a_2(b_1 c_3 - b_3 c_1) + a_3(b_1 c_2 - b_2 c_1) \quad (72)$$

Lines and planes

$$L = a + \lambda.d \tag{73}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} a \\ b \\ c \end{pmatrix} + \lambda \begin{pmatrix} l \\ m \\ n \end{pmatrix} \tag{74}$$

$$\frac{x-a}{l} = \frac{y-b}{m} = \frac{z-c}{n} \tag{75}$$

$$x.n = a.n \tag{76}$$